

Domain Wall Confinement and Deconfinement under Pöschl-Teller Potential on Quantum Simulation

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ABSTRACT

We investigate domain wall confinement and deconfinement through quantum simulation of an effective low-energy Hamiltonian describing the relative motion of a domain wall pair. By mapping the problem onto a discretized Pöschl-Teller potential. Confinement, partial confinement and deconfinement regimes are identified via spatial variance and local probability observables. The close agreement between exact numeric and ideal quantum simulation highlights the emergent single particle description of domain wall physics. Experiments on IBM quantum hardware expose the role of noise and finite-depth dynamics in near-term simulations of non-equilibrium many-body phenomena.

KEYWORDS

Confinement; domain wall; quantum simulation; Pöschl-Teller potential

1 Introduction

Confinement and deconfinement of elementary excitations are central concepts across many areas of modern physics, from quantum chromodynamics to low-dimensional quantum matter^[1-2]. In one-dimensional quantum spin systems, this phenomenon reveals through dynamics of domain walls, as topological defects separating regions of distinct order^[3]. The isolated domain walls behave as free quasiparticles in integrable models, the presence of longitudinal fields or long-range interactions induces an effective attractive force between pairs of domain walls^[4-5]. The confinement in formation of meson-like bound states has been well studied in static and equilibrium. However, recent experimental observations of these effects in trapped-ion systems have stimulated renewed interest in understanding confinement dynamics beyond equilibrium with quantum quench^[5].

Domain wall confinement is not intrinsically magnetic in origin but instead originates from effective interaction between quasiparticles, once the many-body Hilbert space is restricted to a fixed domain wall sector. Here, the complex microscopic spin dynamics collapse into an effective single particle problem, which governs the relative separation of a domain wall pair under a confining potential, with confinement understood in terms of collective degrees of freedom. This theoretical viewpoint provides a natural entry for quantum simulation, as it isolates the essential physics while avoiding unnecessary theoretical microscopic complexity. With Hamiltonian engineering and real-time evolution, we exploit advantage of quantum simulation to study non-equilibrium quantum dynamics in well-controlled environment^[6-7]. Intriguingly, the deconfined dynamics observed display features analogous to deconfined domain wall motion described in magnetic systems, where localized excitations propagate and energy transferred into extended modes. In magnetic nanowires, such deconfinement arises from spin-wave emission and spin-transfer processes^[8-9]. In our effective description, deconfinement here reveals through coupling to continuum eigenstates rather than spin-wave emission, highlighting the universality of deconfined dynamics across distinct physical platforms.

By projecting the full many-body Hamiltonian into the two-domain-wall sector, this yields an effective single-coordinate Schrödinger equation with a confining potential, which we model using the Pöschl-Teller potential that admits analytical intuition and numerical stability. Importantly, the many-body character of our simulation is retained through the exponential Hilbert-space encoding, the non-local Pauli-string Hamiltonian, and the entangling unitary dynamics generated during the Trotterized time evolution. We used Qiskit of IBM quantum platform^[10] to simulate the confinement and deconfinement dynamics of a single effective wavefunction governed by the Hamiltonian ($\hbar = m = 1$)

$$H = -\frac{1}{2}\nabla^2 + V(x) \quad (1)$$

Here, coordinate x represents the relative separation of two domain walls, with potential term $V(x)$ in Pöschl-Teller form that we define later. The kinetic term of our Hamiltonian represents the real space discretization of the one-dimensional Laplacian operator. Correspondingly, $\nabla^2\psi \approx \frac{\psi_{j+1} - 2\psi_j + \psi_{j-1}}{dx^2}$, this is implemented on a uniform spatial lattice with spacing Δx . In this representation, the continuous second derivative is approximated by a second-order central finite difference. The diagonal term proportional to $-2\psi_j$ corresponds to the local curvature contribution at site j , while the off-diagonal terms $\psi_{j\pm 1}$ encode nearest-neighbor hopping between adjacent lattice sites. Physically, this structure reflects the locality of the kinetic energy operator, with momentum fluctuations at given point arise from the finite difference with its immediate neighbors. The resulting kinetic matrix is Hermitian, sparse, and banded, these properties are particularly

useful for mapping our Hamiltonian onto a non-local Pauli-string representation, thus suitable for later variational quantum eigensolvers (VQE) and Trotterization.

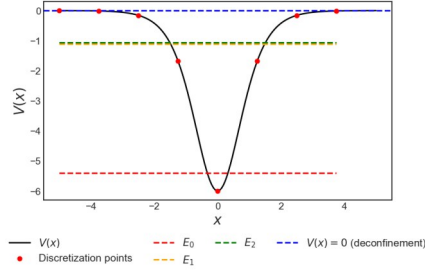


Figure 1

Figure 1. Static confinement spectrum. Pöschl-Teller potential $V(x)$ (black curve) and its bound-state energies E_0 (red dashed line), E_1 (yellow dashed line), E_2 (green dashed line), at potential depth $\lambda=3$. The discretized spatial grid (red markers) used in finite-difference Hamiltonian with $N=2^{3 \text{ qubits}}=8$ points. Deconfinement region postulated at $V(x)=0$ (blue dashed line).

To establish a quantitative reference for confinement in our model, here we analyze the static energy spectrum of the Pöschl-Teller potential (2), which provides an analytically tractable approximation to the confining landscape experienced by a pair of domain walls in meson-like bound states.

$$V(x) = -\frac{1}{2} \lambda(\lambda+1) \text{sech}^2 x \quad (2)$$

The potential $V(x)$ is negative and vanishes asymptotically (Figure 1), such that any discrete eigenstates of the corresponding Schrödinger operator must satisfy $E_n < 0$. These negative eigenenergies constitute the bound-state manifold, representing localization of the domain wall pairs near the center of the potential due to the restoring force generated by the confining interaction. The ground state corresponds to the most negative energy and exhibits maximum spatial localization, reflecting a meson-like tightly bound state, while states that are successively high-lying exhibit increasingly extended wavefunctions and reduced confinement strength. Our potential depth is represented by λ , that we utilized as parameter in quantum quench, a sudden change in our Hamiltonian, to bring the system out of equilibrium. A global quench later increases the effective energy scale of the system, allowing population of the continuum energy, where the potential no longer supports localization. In this regime the domain walls deconfine and propagate, in close analogy with confinement-deconfinement crossover phenomenon observed in the long-range interacting spin chains [5]. Thus, the static Pöschl-Teller spectrum establishes the energetic separation between confined bound states and the unbound continuum, forming the foundation for interpreting dynamical signatures of confinement and deconfinement.

The full Hamiltonian (1) is constructed on an $N=2^n=8$ dimensional Hilbert space, where $n=3$ is the number of qubits we used to encode the spatial lattice. In our simulation, the IBM quantum platform default initial state is $|000\rangle$. The initial quantum state is prepared as the ground state of the pre-quench Hamiltonian, with $\lambda=3$ by potential depth. As a classical benchmark, the ground state is obtained by exact diagonalization of the discretized Hamiltonian, $\hat{H}|\psi_0^{\text{exact}}\rangle = E_0^{\text{exact}}|\psi_0^{\text{exact}}\rangle$. This approach not only provides the exact ground state energy, also serves as a reference for validating the variational quantum approach. To prepare the same ground state on a quantum processor, we employ VQE, $E(\theta) = \langle \psi(\theta) | \hat{H} | \psi(\theta) \rangle$. Our variational parameters θ are optimized by minimizing the energy expectation value, using constrained optimization by linear approximation (COBYLA) until convergence to the minimum eigenvalue E_0^{VQE} . For the confinement strength considered here, the VQE ground state energy is found to closely match the exact result (Figure 2), demonstrating that the variational ansatz successfully captures the localized meson-like ground state, $E_0^{\text{VQE}} \approx E_0^{\text{exact}}$. This agreement confirms that the VQE-prepared state faithfully reproduces the physical properties of the domain wall pair and can therefore be utilized as a reliable initial state for subsequent real-time Trotterization and quantum quench.

To understand the pre-quench confinement dynamics, we study the real-time evolution of the ground state wavepacket in our Pöschl-Teller potential well with fixed depth $\lambda=3$. This provides a stringent test of whether the simulated dynamics preserve localization, as required for a confined phase. The initial state we previously prepared using VQE have subsequently evolved under the same static Hamiltonian using a first-order Suzuki-Trotter decomposition (3).

$$e^{-iHt} \approx \left(e^{-iT\Delta t} e^{-iV\Delta t} \right)^{n_{\text{steps}}} + \mathcal{O}(\Delta t^2) \quad (3)$$

Figure 2. Comparison between exact and VQE ground state energies for the discretized Pöschl-Teller potential. The exact value is obtained by classical diagonalization of the 8-dimensional finite-difference Hamiltonian. The VQE result uses a 3-qubit Pauli encoded Hamiltonian. The small deviation (*absolute error* < 0.04) demonstrates that VQE faithfully reproduces the lowest bound state of confining potential.

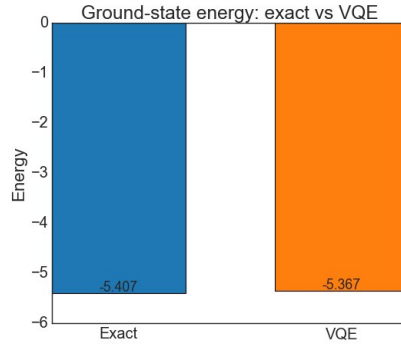


Figure 2

In this case, $t = n_{\text{steps}} \Delta t$, that n_{steps} marks the number of Trotter steps, $\mathcal{O}(\Delta t^2 \| [T, V] \|)$ gives Trotter errors in commutator, with T and V representing our kinetic and potential terms respectively. Here, we monitor two complementary observables, the spatial variance $\sigma_x(t)$ and the center-site return probability $P_0(t)$.

$$\sigma_x(t) = \sqrt{\langle x^2 \rangle_t - \langle x \rangle_t^2} \quad (4)$$

$$P_0(t) = |\langle x=0 | \psi(t) \rangle|^2 \quad (5)$$

The spatial variance quantifies the width of the wavepacket, while the center-site return probability measures localization at the minimum of the confining potential. Theoretically, for the exact ground state of a time-independent Hamiltonian, both quantities should remain constant in time, reflecting the stationarity of the eigenstate. As expected, the exact numerical evolution exhibits time-independent values for both $\sigma_x(t)$ (Figure 3a) and $P_0(t)$ (Figure 3b), indicated by the dashed black reference line. In contrast, the Trotterized quantum simulation displays small amplitude sinusoidal oscillations in both observables. These oscillations originate from coherent interference between the dominant ground state component and weakly populated excited bound states. Since our VQE prepared initial state is not exactly the true ground state $|\psi_0\rangle$, but a very high-fidelity approximation.

$$|\psi_{\text{VQE}}\rangle = c_0 |\psi_0\rangle + \sum_{n>0} c_n |\psi_n\rangle \quad (6)$$

With probability of measured ground state $|c_0|^2 \approx 1$, and pre-quench time-independent Hamiltonian, our energy eigenstate acquires only a phase. Our observables, either spatial variance or return probability represented in $\langle O(t) \rangle$ therefore entails bound state interference terms.

$$\langle O(t) \rangle = \sum_{m,n} c_m^* c_n e^{i(E_m - E_n)t} \langle \psi_m | O | \psi_n \rangle \quad (7)$$

Hence our dynamics are governed by a single dominant energy difference corresponding to oscillation frequency in this case, with E_1 the lowest excited eigenenergy that mostly contributes, $\omega_b = E_1 - E_0$, leading to purely sinusoidal oscillations and correspondingly a breathing mode. Meanwhile, Trotter errors remain small due to our choice of smooth spatial variation of the Pöschl-Teller potential and strictly local structure of the kinetic term.

However, their amplitude remains bounded over the entire evolution window, and no secular growth is observed. Crucially, the absence of any long-time increase in $\sigma_x(t)$, together embracing the persistence of high center-site return probability, demonstrates that the wavepacket remains spatially confined throughout the evolution. Without spreading into any extended states here, we confirm that the observed dynamics are attributed to a confined regime rather than diffusive or ballistic propagation. This static confinement specification establishes a reference against which the post-quench deconfinement dynamics can be apparently identified.

We now investigate the real-time dynamics following a sudden quantum quench of the confining potential. In this project, we probe these dynamics using both ideal quantum simulation in noise-free condition, classical exact numeric, and real IBM quantum hardware, allowing a direct comparison between theory and experimental-level behavior. The quantum quench is implemented by preparing the system in the ground state of an initial Hamiltonian H_i that we previously demonstrated and subsequently evolve under a distinct post-quench Hamiltonian H_f . In our case, the pre-quench Hamiltonian corresponds to a deep Pöschl-Teller potential with $\lambda_i = 3$, while the post-quench Hamiltonian is characterized by a reduced potential depth $\lambda_f = 1$ and $\lambda_f = 0$, correspond respectively a partially confining region and a fully deconfined regime. This allows us with controlled tuning between confining and deconfining regimes. Beyond local observables, here we monitor the relative growth in spatial variance (8), which quantifies the spreading of wavepacket from its initial position at time $t = 0$ (Figure 4).

$$\Delta \sigma_x(t) = \sigma_x(t) - \sigma_x(0) \quad (8)$$

Since quantum quench dynamics are governed by phase accumulation between post-quench energy eigenstates, our observables entailed the same physical structure as in the previous interference term (7). When we quench to $\lambda_f = 1$ at the

partially confining regime, the previously mentioned E_1 as lowest excited eigenenergy does not disappear, even the Pöschl-Teller potential now supports only a single ground bound state. Rather, it ceases to be a bound state excitation and becomes a continuum scattering excitation. This originates from our simulation is performed on a finite discretized lattice with $N=2^n$ sites, and energy as discretized continuum eigenstate $E \in \{E_1, E_2, \dots, E_{n-1}\}$, thus $H_f|\psi_m\rangle = E_m|\psi_m\rangle$ still holds. In this partially confining regime, there exist nearby E_m as a set of low energy continuum modes, therefore interference term leads to phase accumulation and relatively small wavepacket spreading (Figure 4a-c). At this point, the wavepacket dynamics represent partial confinement characteristics, the spatial variance initially increases with following expansion arrested by the residual confining potential. This behavior reflects the persistence of an effective confining interaction that limits the separation of the domain wall pair.

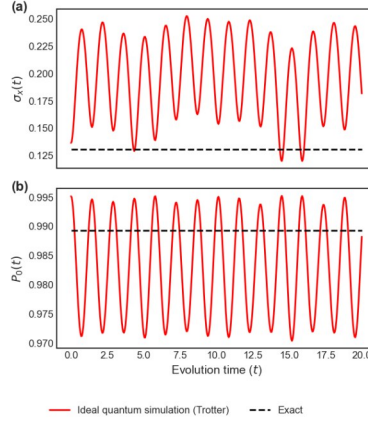


Figure 3

Figure 3. Pre-quench static confinement in Pöschl-Teller potential of potential depth $\lambda=3$. Time evolution of spatial variance $\sigma_x(t)$ (a) and center-site return probability $P_0(t)$ (b). Initial ground state wavepacket prepared via VQE (red curve) shows ideal first-order Suzuki-Trotter quantum simulation with $n_{\text{step}}=2$, compared to exact numerical ground-state values (black dashed line).

As the post-quench potential depth is set to $\lambda_f=0$, the Pöschl-Teller potential vanishes entirely and the system becomes governed solely by the kinetic operator, $H_f=T$. Physically, this corresponds to a fully deconfined regime, in which no bound state exists, and domain walls propagate freely without an effective confining force. The Hamiltonian here possesses a continuous energy spectrum corresponding free particle scattering state, derived through discrete dispersion relation, $H_f|\psi_k\rangle = \frac{k^2}{2}|\psi_k\rangle$. Importantly, the absence of bound states does not imply the absence of energy eigenvalues in the time evolution. As mentioned before, our system is represented on a finite real space grid and encode using a finite number of qubits, therefore wavepacket cannot expand beyond the system size. This discretization converts the continuum into a dense but discrete set of eigenvalues, which approximate the momentum eigenstates. These states now represent discretized scattering modes of the deconfined Hamiltonian. Immediately after the quench, the initial state $|\psi_0\rangle$, prepared as the ground bound state of the pre-quench Hamiltonian, is no longer an eigenstate of H_f . Instead, it decomposes as,

$$|\psi_0\rangle = \sum_m c_m |\psi_m^f\rangle \quad (9)$$

where $|\psi_m^f\rangle$ are eigenstates of the post-quench free Hamiltonian, with expansion coefficients $c_m = \langle \psi_m^f | \psi_0 \rangle$. The time evolution here follows,

$$|\psi(t)\rangle = \sum_m c_m e^{-iE_m t} |\psi_m^f\rangle \quad (10)$$

At this point the energies E_m are attributed to a quasi-continuous set, where the phase factors $e^{-iE_m t}$ rapidly dephase due to dense energy spacing. Although the larger scaled relative growth of spatial variance exhibits non-monotonic tilts (Figure 4d-f), these do not signal residual confinement. Instead, they arise from coherent interference among dispersive kinetic eigenmodes in our finite system. The rapid early growth of $\Delta\sigma_x(t)$ reflects ballistic transport character at early times, with a broad maximum reached around time $t=8\sim 10$. The disappearance of bound state and the emergence of a quasi-continuous spectrum led to dynamical unbinding of the meson-like domain wall pair, analogous to domain wall propagation in comparative magnetic systems [8-9]. At later times, the finite system size and coherent interference led to saturation behavior, revealing the closed and unitary nature of dynamics rather than residual confinement.

In complementation we examine the center-site return probability (5), which is sensitive to local probability refocusing and therefore provides a rigorous analysis of partial confinement and deconfinement. In (Figure 5), dashed horizontal lines indicate the temporal average of $P_0(t)$, highlighting the long-time redistribution of probability within the finite lattice. For the partially confining region, the ideal Trotterized quantum simulation exhibits persistent but bounded

oscillation of $P_0(t)$ around a finite mean value (Figure 5a-c). That is significantly above the uniform background probability value for $N = 8$ lattice sites, $P_0 = 1/N$, the value expected for a completely delocalized wavefunction on a finite lattice. These oscillations arise from phase accumulation between the bound ground state and nearby continuum eigenstates. Significantly, since the bound state remains energetically isolated, probability weight repeatedly refocuses near the center, preventing full delocalization. In the deconfined regime, the post-quench Hamiltonian consists of no bound state, with all eigenstates in the continuum. The initial wavepacket therefore evolves as a superposition of extended modes driving to rapid redistribution of probability away from the center. The ideal quantum simulation here shows a prominent initial decay of $P_0(t)$ through the uniform background value, signaling the loss of localization at the origin (Figure 5d-f). However, this decay does not proceed to zero, instead $P_0(t)$ saturates at a small but finite value and exhibits residual fluctuations. This saturation is fully consistent with the spatial variance results (Figure 4d-f), where $\Delta\sigma_x(t)$ grows rapidly but subsequently levels off due to finite Hilbert space dimensionality. Thus, the deconfined regime here should be understood as effective deconfinement rather than asymptotic free expansion in an infinite system.

The close agreement between the exact time evolution and the ideal Trotterized quantum simulation arises from the fact that both approaches implement the same finite-dimensional post-quench Hamiltonian, differing only in the numerical strategy utilized to generate the time evolution. However, the Trotter error term (3) in commutator form remains small due to our selection of both potential and kinetic operator as mentioned earlier. Moreover, in the partially confining and deconfined regimes, the observed wavepacket spreading is dominated not by numerical errors but physical dephasing originated from interference between many closely spaced continuum eigenmodes. This intrinsic spectral broadening sets an energy and time scale that is much larger than any residual Trotter error. As a result, the exact and ideal Trotterized dynamics become practically indistinguishable, which is confirmed by their independence of increments in number of Trotter steps (Figure 4-5). This agreement provides validation of the Trotterized quantum simulation framework in capturing non-equilibrium confinement and deconfinement dynamics.

In contrast, the behavior observed on real quantum hardware (Figure 4-5) shows an obvious dependence on the Trotter step number. For both observables, increasing n_{steps} leads to a systematic improvement of agreement with the ideal and exact results. At low Trotter depth of $n_{\text{steps}} = 2$, the hardware results demonstrate a pronounced enhancement of the return probability at intermediate time $t = 8 \sim 10$, forming a broad maximum that exceeds both ideal and exact predictions (Figure 5a and 5d). The feature here is not simply attributable to decoherence or noise-induced damping, as such effects would suppress coherent oscillations and drive the probability toward uniform background value. Instead, this enhancement arises from the interplay between coherent quantum dynamics and circuit-level imperfections at shallow depth. With few Trotter steps, the implemented unitary contains relatively large discrete time steps, amplifying phase errors associated with coherent over rotations and gate dependent control imperfections. These errors can transiently deflect probability flow back toward the center site, effectively over-localizing the wavefunction at intermediate times. This behavior consists of a coherent misrepresentation of the effective Hamiltonian, rather than incoherent loss of quantum information. As n_{steps} increases, the Trotter step size is reduced, the accumulated phase error per step diminishes, and the hardware dynamics relax toward the ideal behavior. Here the premature saturation (Figure 4d-f) and reduced probability density amplitude (Figure 5) of hardware results are affected by residual decoherence and readout noise in near term quantum devices. While the vanishing center-site probability observed on hardware at $t = 0$ originates from nonlocal encoding of position states across the qubit register, this offset does not affect subsequent dynamical signatures of partial confinement and effective deconfinement, which are encoded in temporal evolution rather than absolute normalization.

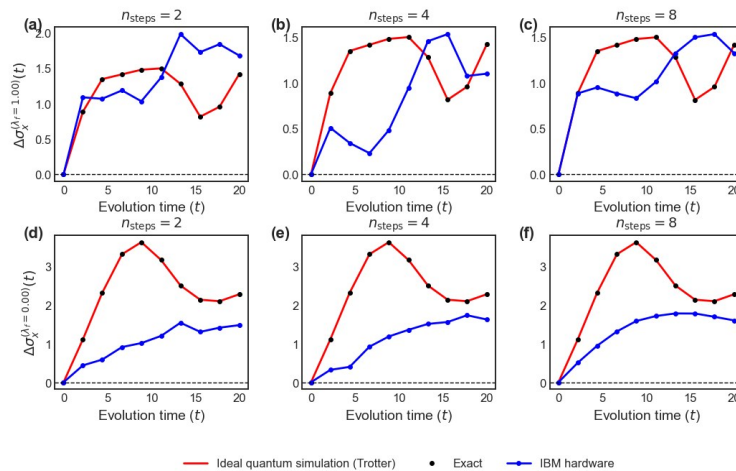


Figure 4

Figure 4. Post-quench wavepacket spreading across partially confined and deconfined regimes. Time evolution of relative growth in spatial variance $\Delta\sigma_x(t)$ following a quantum quench from the ground state. (a-c) show the partially confined regime $\lambda_f=1$, (d-f) show the deconfined regime $\lambda_f=0$, for increasing Trotter step numbers $n_{\text{steps}}=2, 4, 8$. Demonstration of ideal Trotterized quantum simulation (red curve), exact numerical evolution (black marker), and results obtained on IBM quantum hardware (blue curve). Hardware results obtained through IBM Torino.

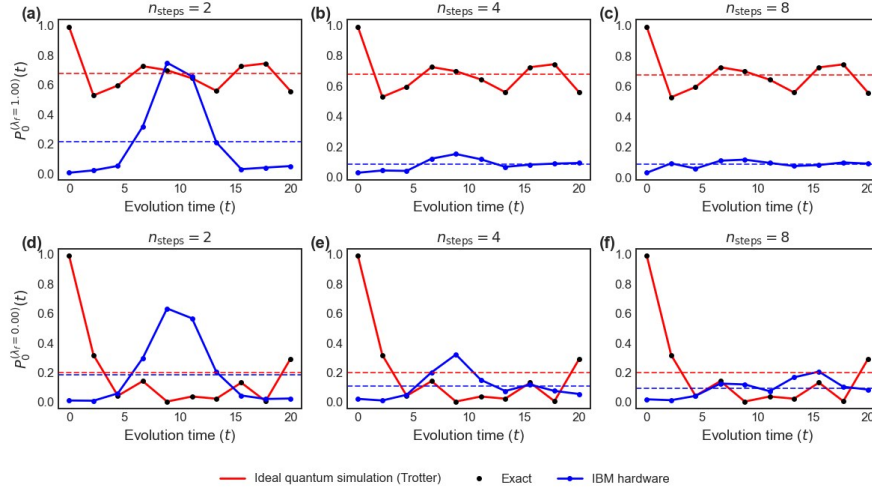


Figure 5

Figure 5. Post-quench center-site return probability across partially confined and deconfined regimes. Time evolution of the center-site probability $P_0(t)$ following a quantum quench from the ground state. (a-c) show the partially confined regime $\lambda_f=1$, (d-f) show the deconfined regime $\lambda_f=0$, for increasing Trotter step numbers $n_{\text{steps}}=2, 4, 8$. Demonstration of ideal Trotterized quantum simulation (red curve), exact numerical evolution (black marker), results obtained on IBM quantum hardware (blue curve) and respectively time-averaged return probability (dashed horizontal lines).

In summary, we have presented a digital quantum simulation of confinement and deconfinement dynamics of domain wall using a discretized continuum model with tunable Pöschl-Teller potential. By combining variational ground state preparation with Trotterized real time evolution, we directly probed bound, partially confined, and deconfined regimes following a quantum quench. The observed breathing dynamics, saturation effects, and delocalization behavior proved clear dynamical signatures of confinement and string breaking. Agreement between ideal quantum simulation and exact numeric, together with qualitative consistency on quantum hardware, demonstrates the viability of near-term quantum devices for exploring non-equilibrium confinement phenomena in many-body quantum systems.

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